



# Feynman-alpha and covariance-to-mean methods for reactivity estimation in a SUR100 research reactor

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# Introduction

- Subcritical measurements were performed in the RA-4 teaching reactor (SUR100)
- Application of  $\alpha$ -Feynman and covariance-to-mean for the estimation of kinetic parameters: reactivity, prompt decay constant and reactor power
- Improvement in the implementation of the methods in order to reduce correlation in data
- Reactivity of some configurations were compared with another method based on the inverse kinetic equations
- These measurements will serve as a reference value to validate simulations of reactor noise experiments with Monte Carlo neutron code (MCNP6.2)



# RA-4 reactor



- The RA-4 (Argentinean Reactor 4) is a Siemens-Unterrichts-Reaktor of nominal power of 100 mW (SUR100)
- It is now licensed to operate up to a maximum continuous power of 1 W

- It is located in the main campus of the National University of Rosario (UNR), Santa Fe Province.

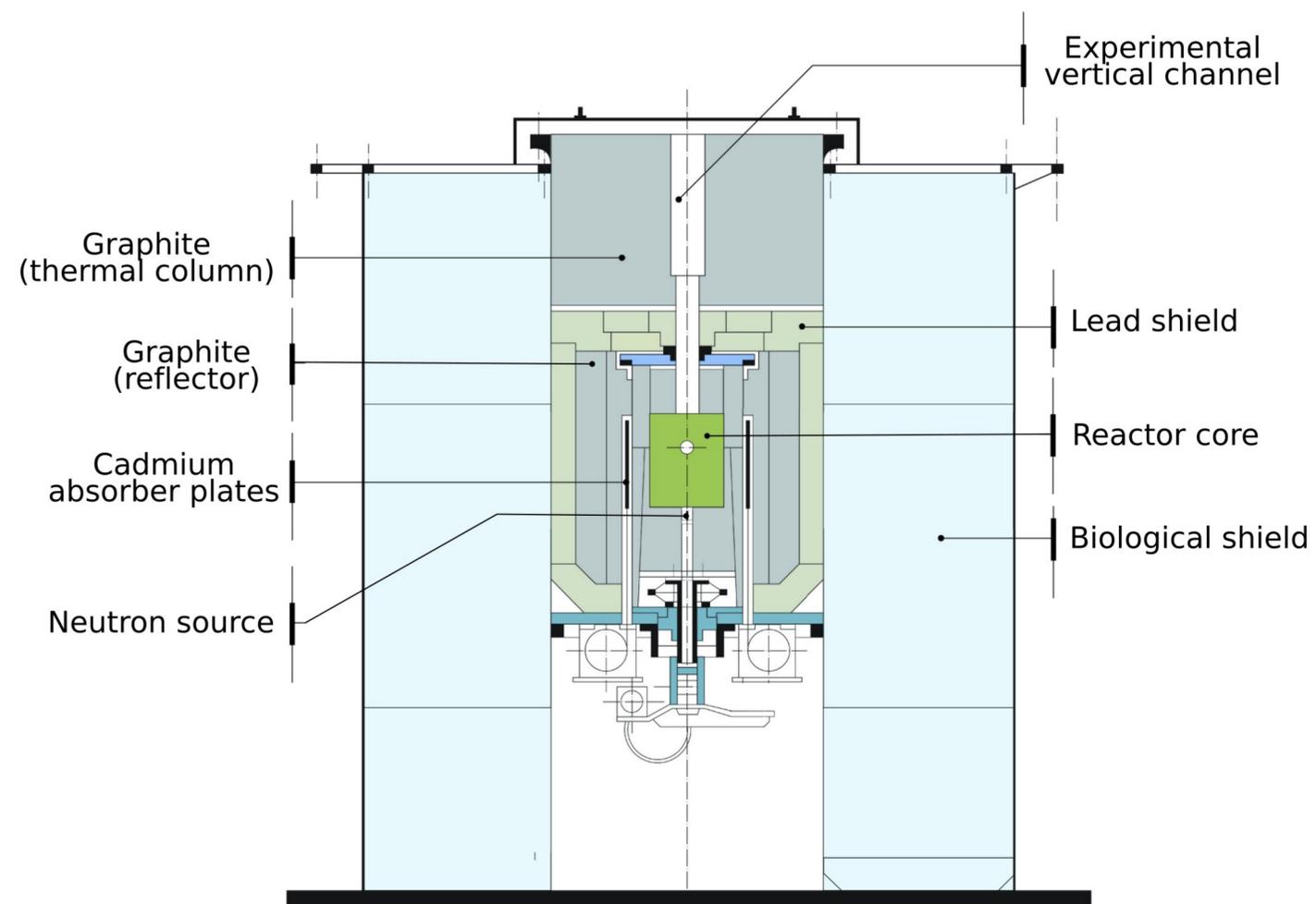


- The reactor is owned by the CNEA and the UNR is the primary responsible of its operation

- Its main purpose is educations and training (students, NPP operators, technicians)
- Research activities from different fields: physics, electronics, control systems, material science, etc.

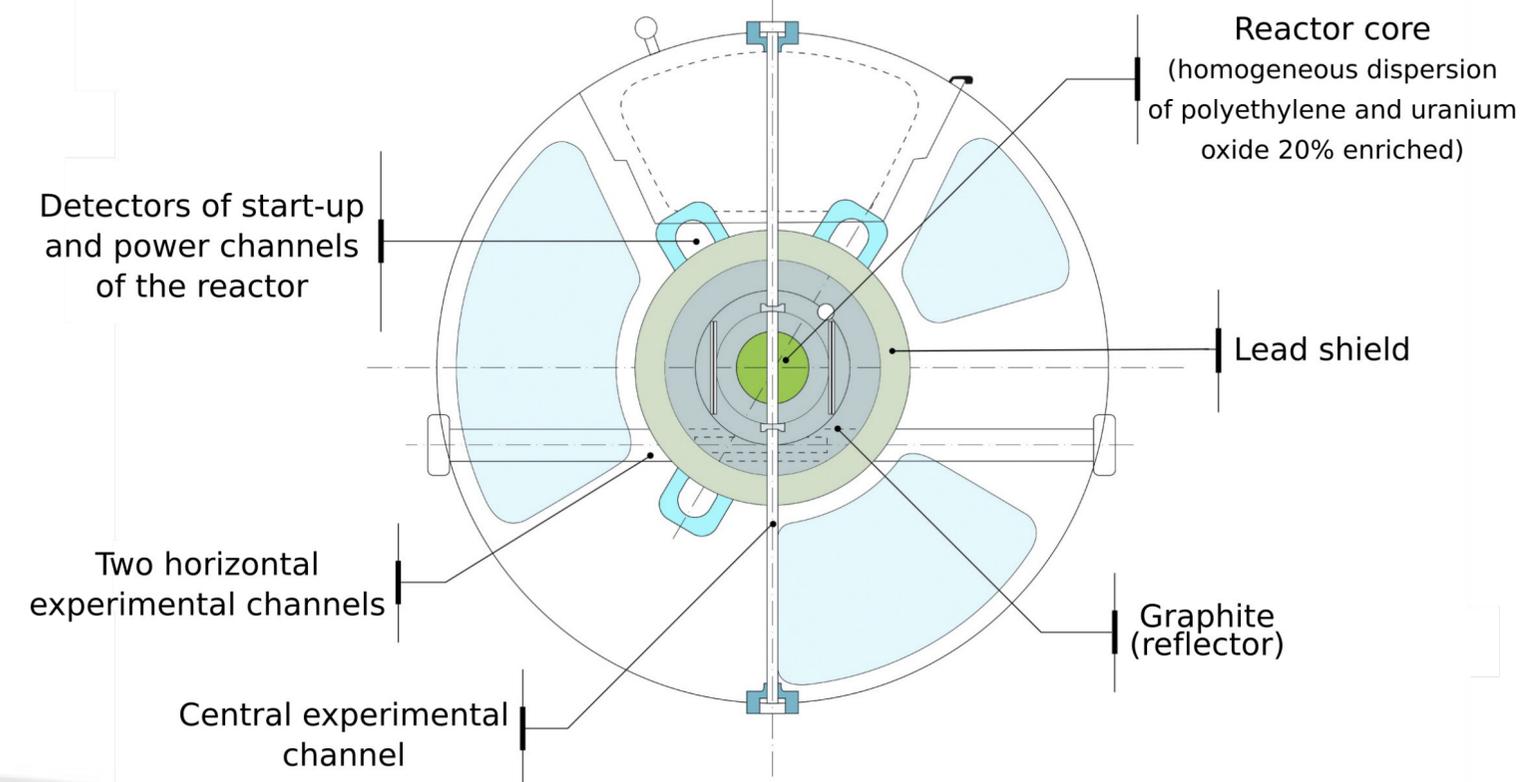


# RA-4 reactor



- One central experimental channel crossing the core
- Two horizontal channels (tangential) and one vertical channel coaxial with the core (thermal column)
- One start-up channel ( $\text{BF}_3$ ) and two power range channel (CIC)

- Solid core with cylindrical shape (24 cm diameter and 26 cm height) divided in two separable sections
- The lower section is movable up to 4.7 cm from the upper section (shutdown condition)
- The fuel is a homogeneous dispersion of  $\text{U}_3\text{O}_8$  (19.86% enriched) and polyethylene
- Average thermal flux:  $3.4 \cdot 10^7 \text{ n}/(\text{cm}^2 \text{ s})$
- Two cadmium absorber plates moving vertically in the reflector
- Cylindrical graphite reflector with a thickness of 20 cm



# $\alpha$ -Feynman method

- The method was developed by Richard Feynman in the 40's during the Manhattan Project
- Allows the determination of the prompt neutron constant ( $\alpha$ ) and fission rates at subcritical stationary states
- It is based on the statistical analysis of neutron counts in a certain time interval  $\tau$
- Due to the presence of a multiplying medium, the variance is greater than would be in a Poisson process:

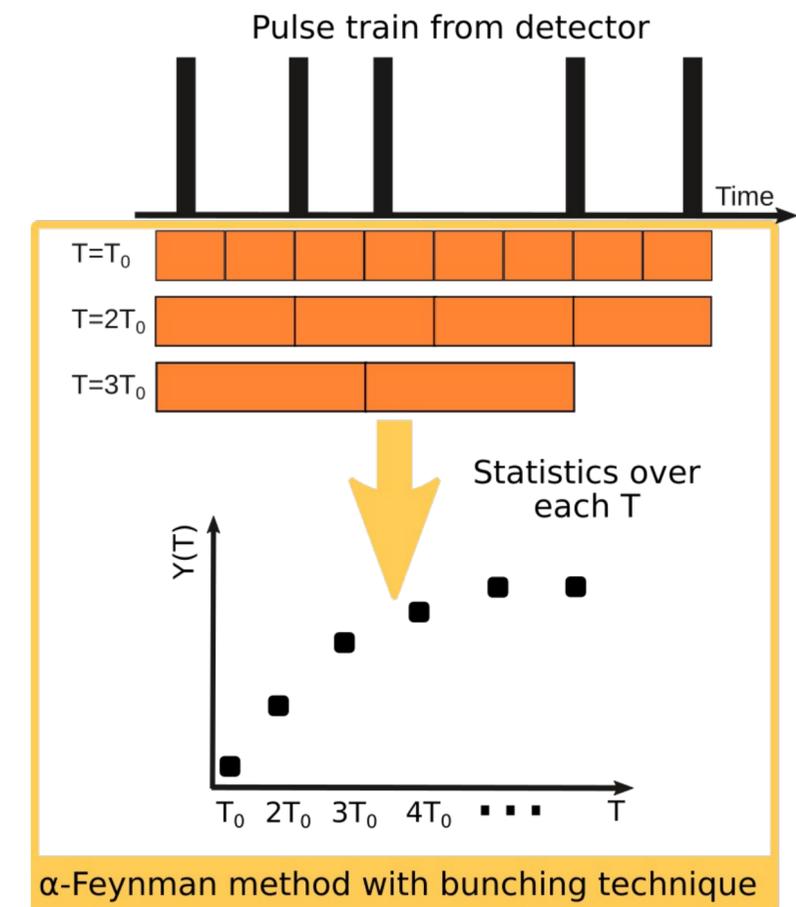
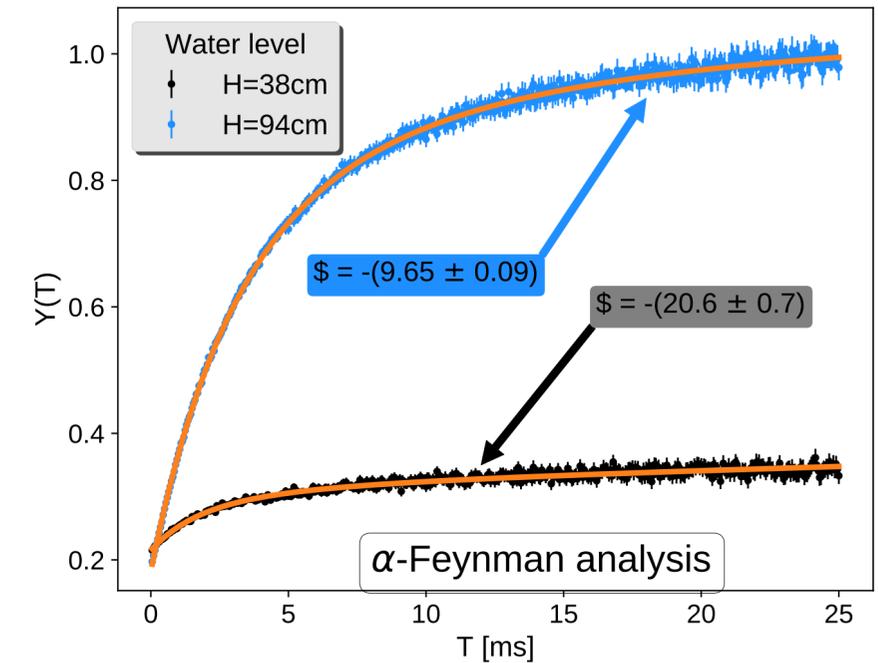
$$Y(\tau) = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle} - 1 = \frac{\epsilon D}{\alpha^2 \Lambda^2} \left( 1 - \frac{1 - e^{-\alpha\tau}}{\alpha\tau} \right) - 2Rd$$

- If instead of the variance, the covariance from two detectors is computed, then we obtain the covariance-to-mean ratio:

$$\text{Cov-to-mean}(\tau) = \frac{\langle N_1 N_2 \rangle - \langle N_1 \rangle \langle N_2 \rangle}{\sqrt{\langle N_1 \rangle \langle N_2 \rangle}} = \frac{\sqrt{\epsilon_1 \epsilon_2} D}{\alpha^2 \Lambda^2} \left( 1 - \frac{1 - e^{-\alpha\tau}}{\alpha\tau} \right)$$

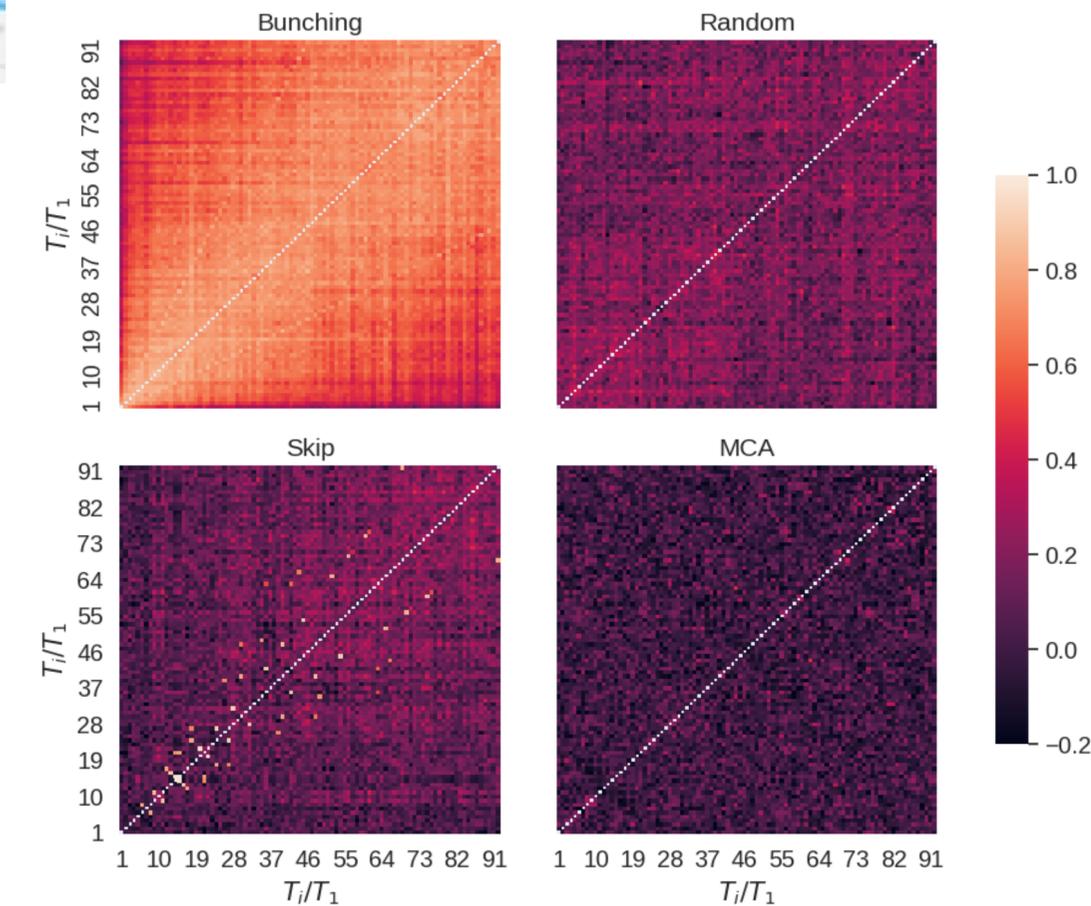
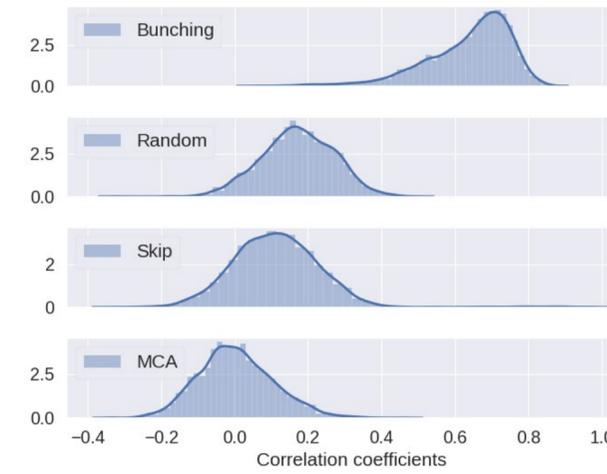
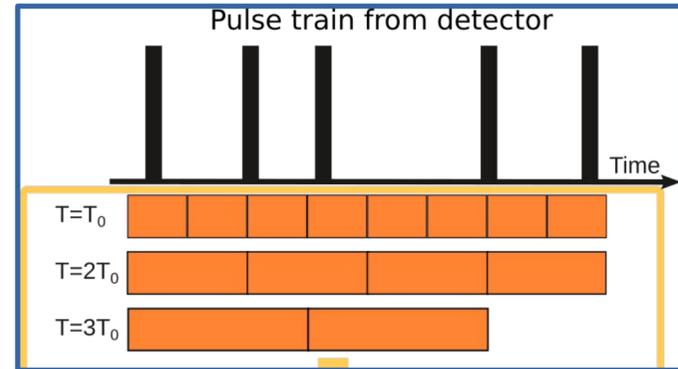
- In order to reduce the measurement time, the  $\alpha$ -Feynman method is usually implemented with the bunching technique:

- Acquisition is made in a fixed time windows  $T_0$ .
- The rest of the  $\tau$  are obtained bunching the original  $T_0$  interval



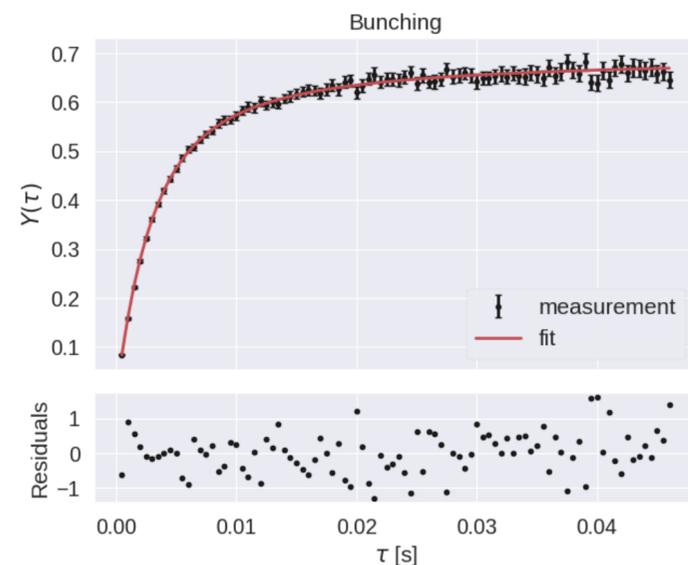
# Reducing the data correlation

- The bunching technique produces highly correlated data that in turn, produce an overestimation of the uncertainties at each point of the  $\alpha$ -Feynman curve



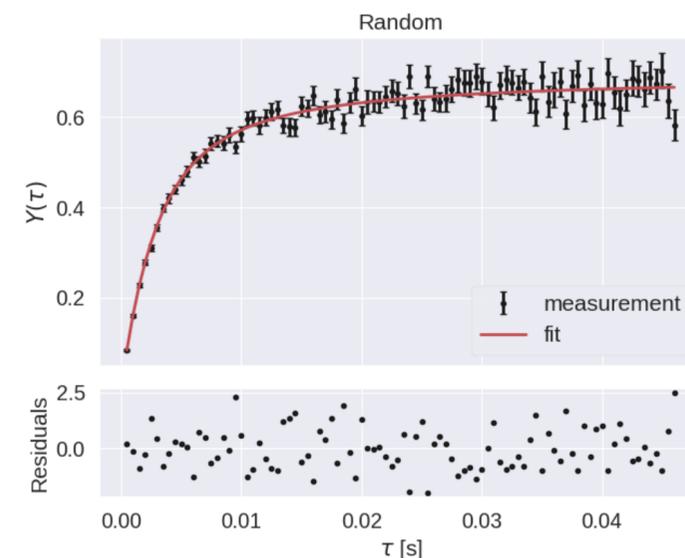
- The random sampling takes a random numbers of intervals at each  $\tau$  (25 %). It was found to be a good trade off between correlation and uncertainties of the curve

## $\alpha$ -Feynman method using some variations from the bunching technique



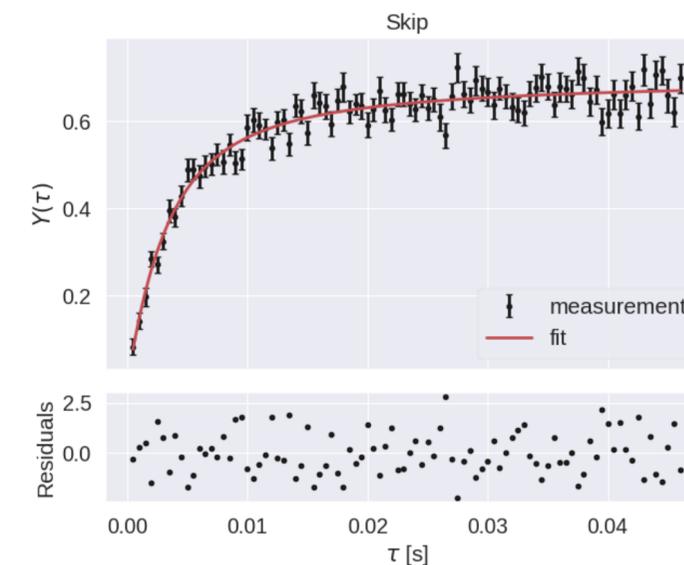
$$\alpha = 572 (5) \text{ 1/s}$$

$$X^2 / \text{dof} = 0.366$$



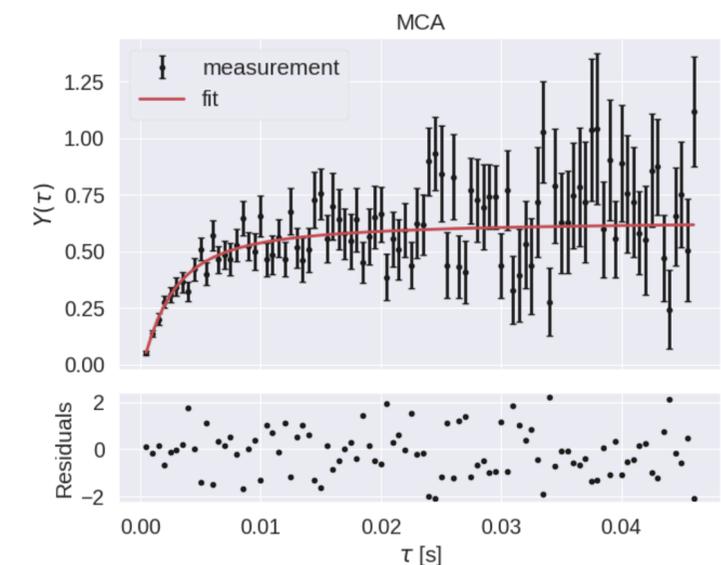
$$\alpha = 575 (10) \text{ 1/s}$$

$$X^2 / \text{dof} = 0.890$$



$$\alpha = 508 (40) \text{ 1/s}$$

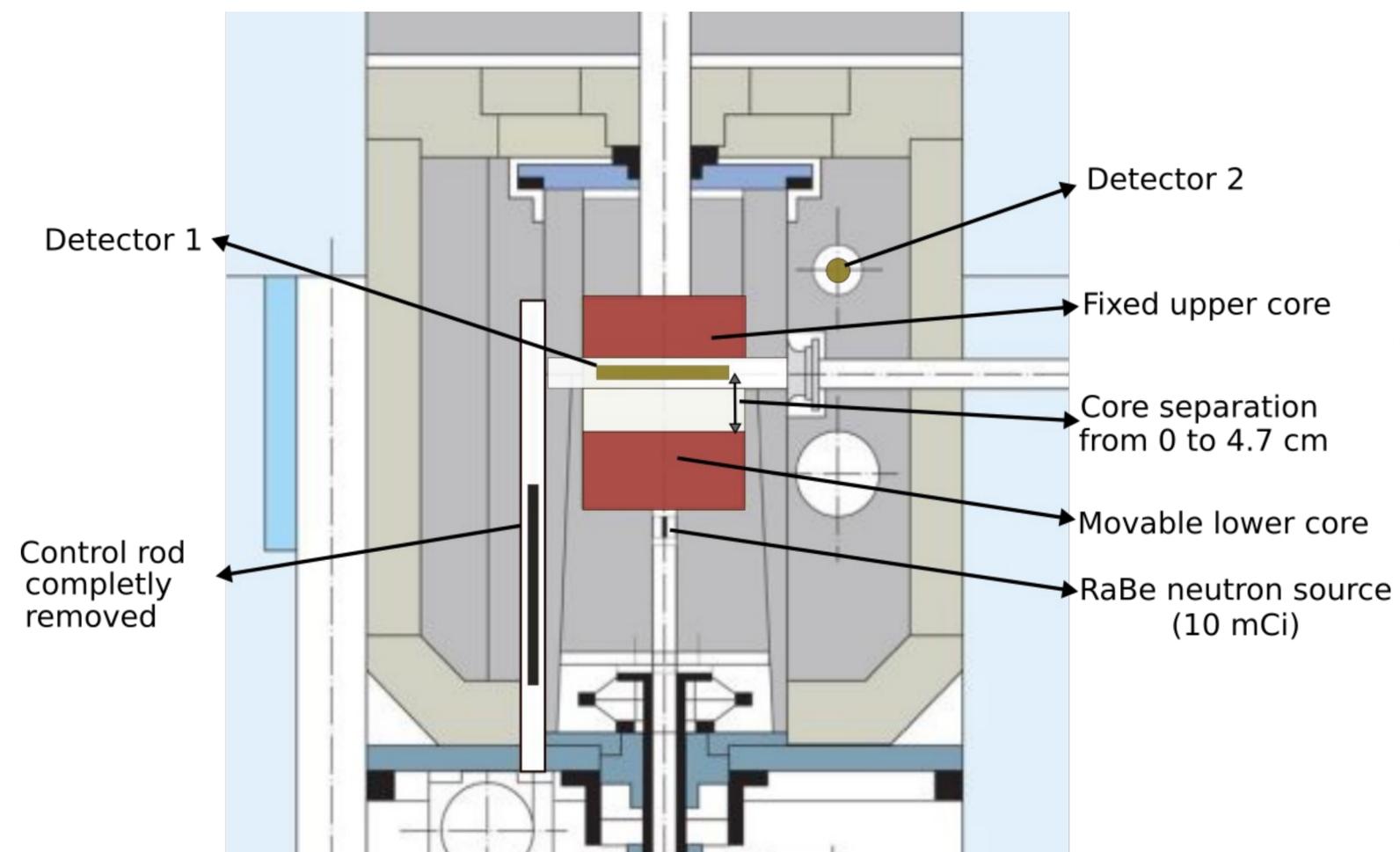
$$X^2 / \text{dof} = 1.114$$



$$\alpha = 672 (90) \text{ 1/s}$$

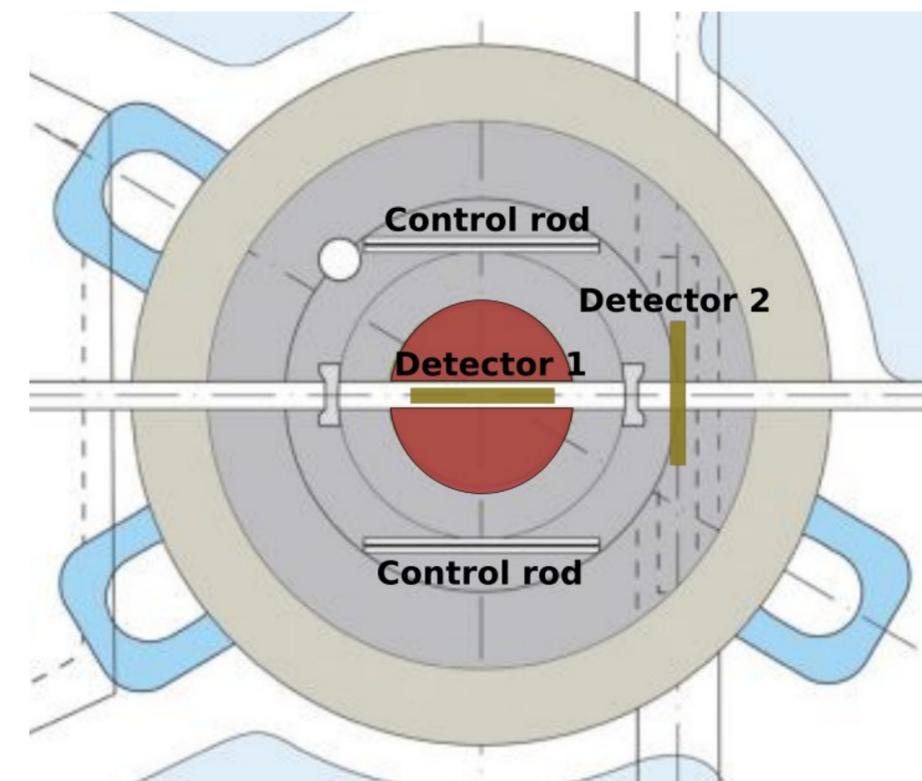
$$X^2 / \text{dof} = 1.027$$

# Experiment description



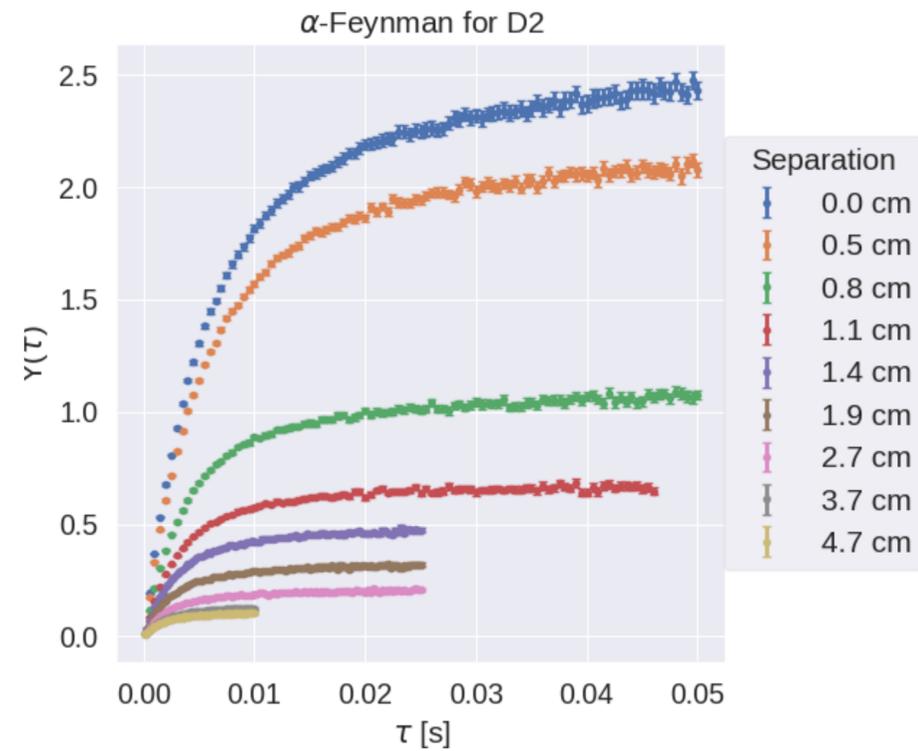
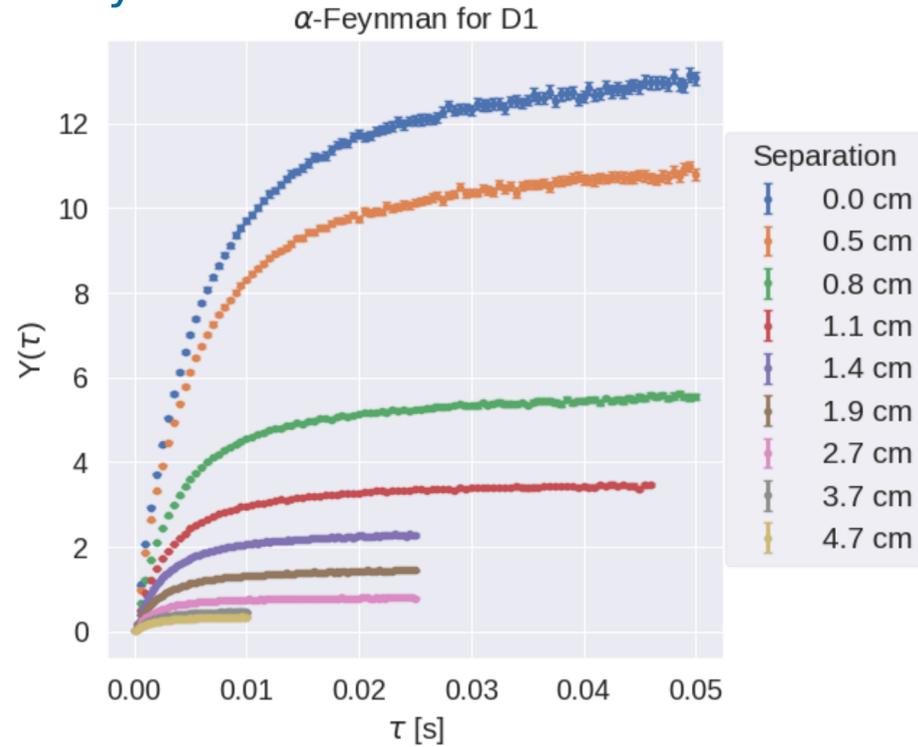
- Two  $^3\text{He}$  neutron detectors (with sensitivity of 6 cps/nv), one located in the core region and the other in the reflector
- Reactor neutron source of Ra-Be with activity of 10 mCi ( $\sim 1.5 \cdot 10^5$  n/s)
- The two control plates remained fully extracted during the measurements
- Measurements were performed at nine core separation distances
- The reactor could not achieve criticality with this detectors configuration

- Each measurement consisted in registering detected pulses in time windows of  $50 \mu\text{s}$
- The measurement time at each configuration took approximately 30 minutes
- Due to high count rates, listmode acquisition was not possible ( $\alpha$ -Rossi method was excluded)



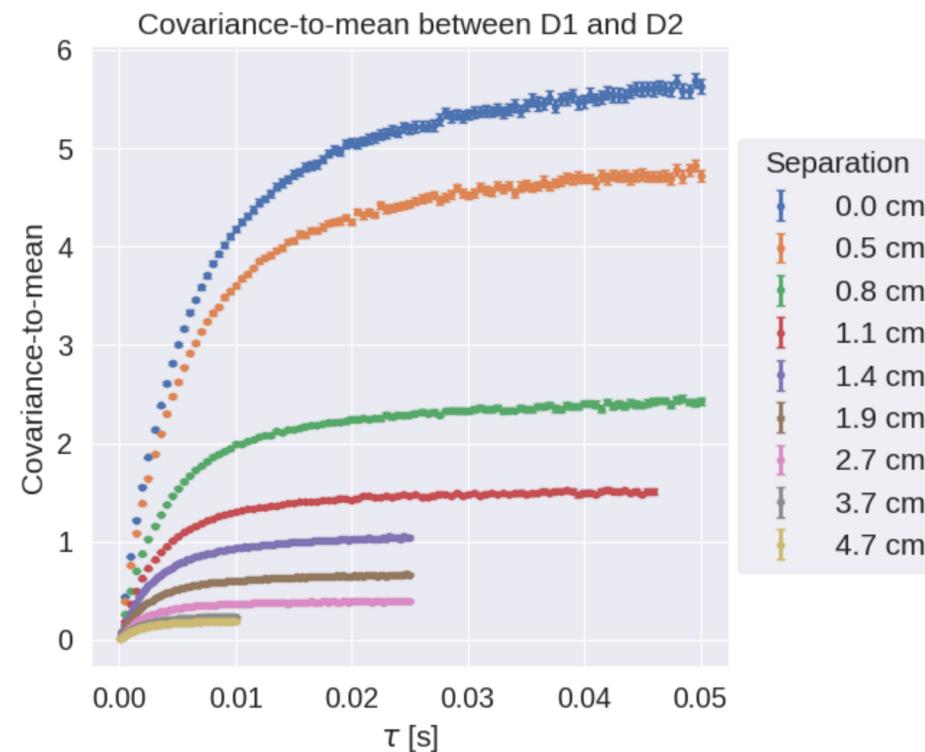
# Results

- $\alpha$ -Feynman curves for each detector



- Covariance-to-mean curves

No significant improvement as the dead time ( $\sim 1\mu\text{s}$ ) is low compared to the low count rates and high detector efficiencies



## Summary of the main features of each level

d [cm]	$R_1$ [kcps]	$R_2$ [kcps]	$\alpha$ [1/s]	p [mW]
0.0	18.5	3.5	322(3)	0.129(3)
0.5	15.4	2.9	336(4)	0.115(3)
0.8	9.1	1.8	472(7)	0.070(2)
1.1	6.4	1.3	575(14)	0.055(3)
1.4	5.0	1.0	637(12)	0.051(2)
1.9	3.7	0.81	753(22)	0.043(2)
2.7	2.6	0.62	831(30)	0.042(3)
3.7	1.9	0.49	902(32)	0.044(3)
4.7	1.6	0.44	975(44)	0.040(3)

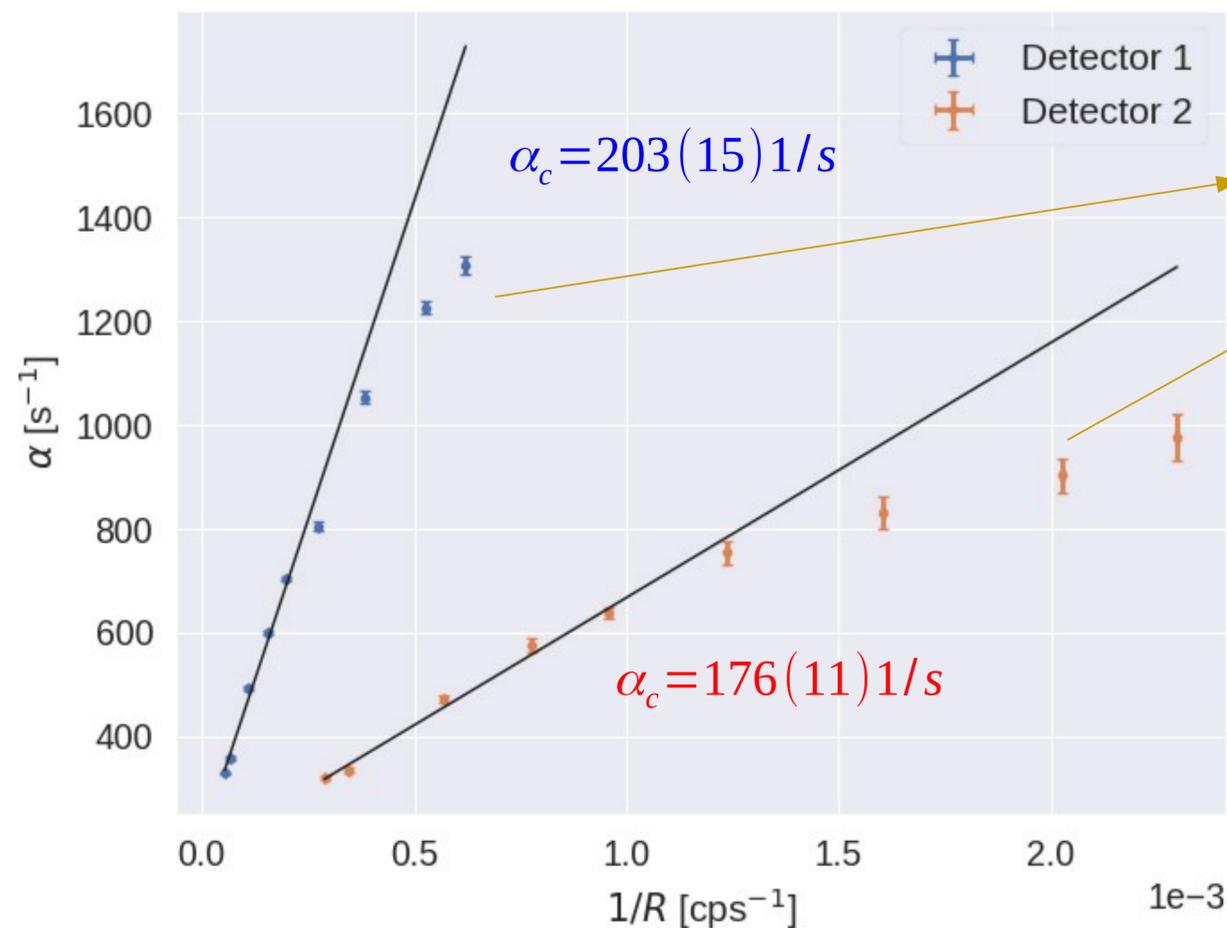


# Reactivity estimation

- It is possible to extrapolate the data to obtain the prompt neutron decay constant at critical state  $\alpha_c$

$$\alpha = \frac{\tilde{Q}}{R} + \alpha_c$$

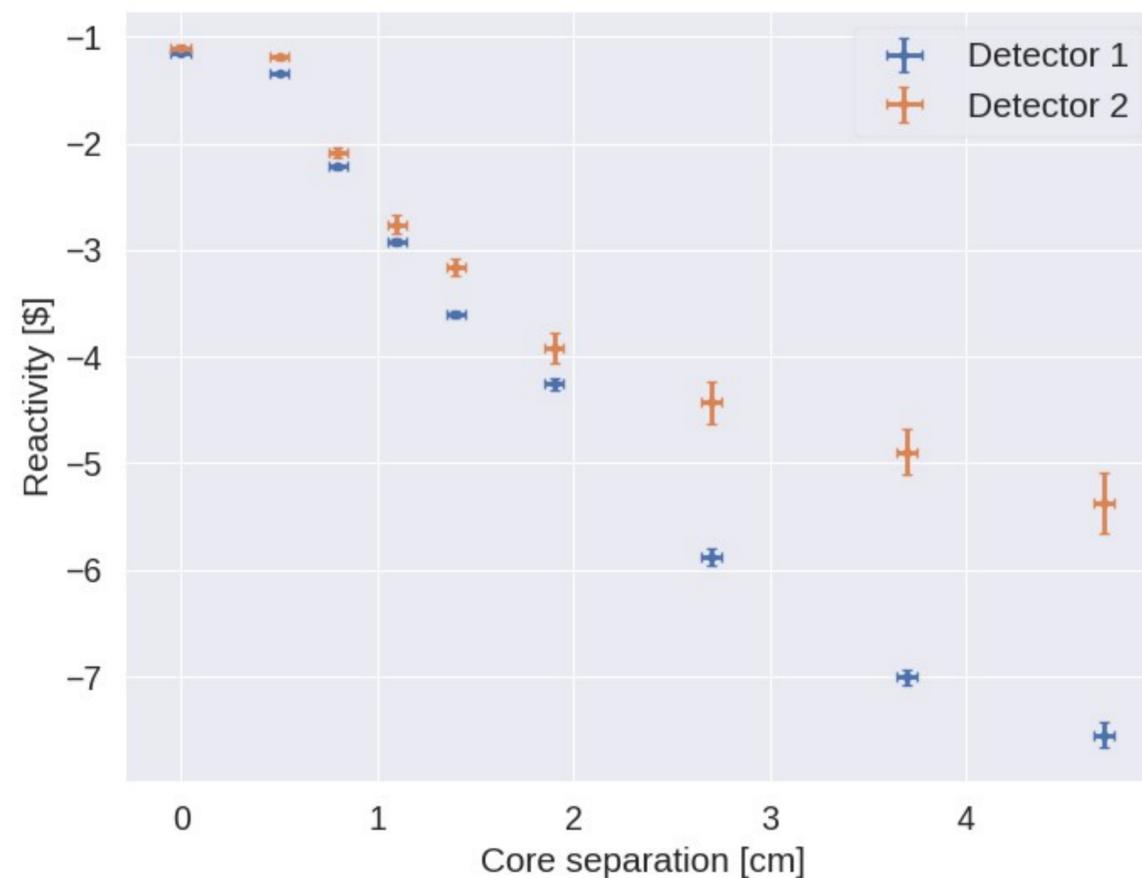
$\tilde{Q}$  is the effective neutron source strength (detector dependant)



Measured at critical state with a CIC (in the reflector) using spectral analysis:  
 $\alpha_c = 153 (2) 1/s$

- The reactivity of each state can be calculated using the relation:

$$\$ = 1 - \frac{\alpha}{\alpha_c}$$

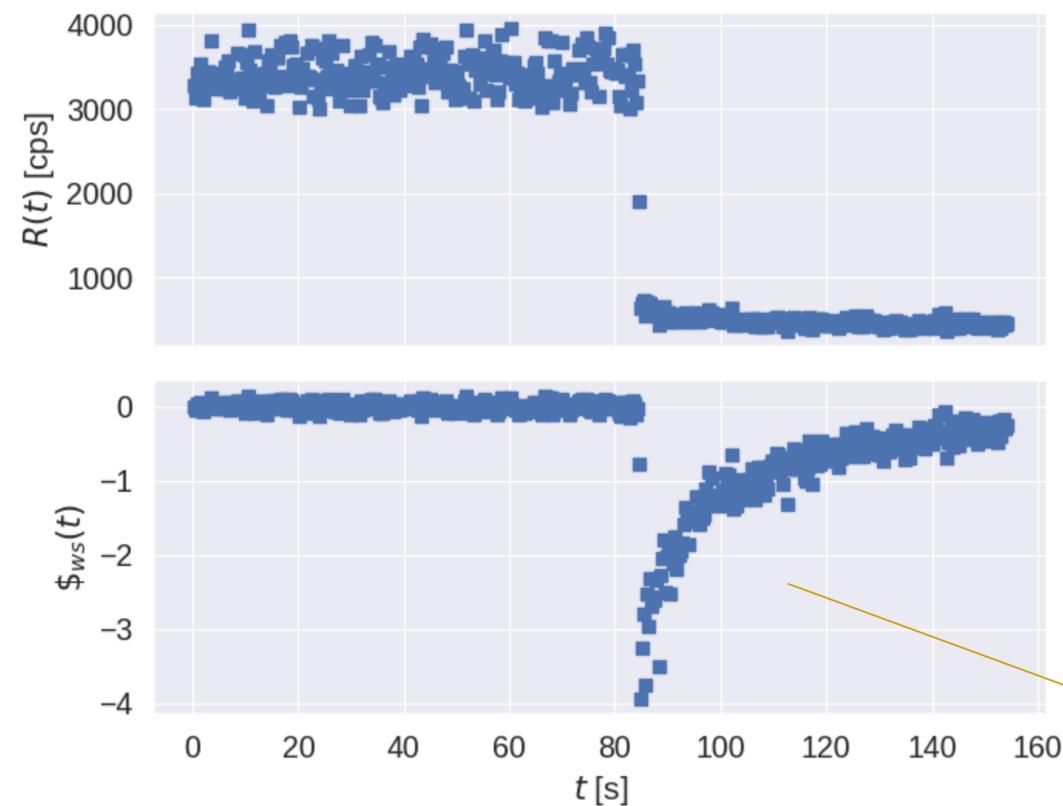


Reactivity worth of the lower part of the core:

$\$ = 6.4(1)$  Detector 1  
 $\$ = 4.3(3)$  Detector 2  
 $\$ = 6.83$  Reference value

# Reactivity worth of the lower core

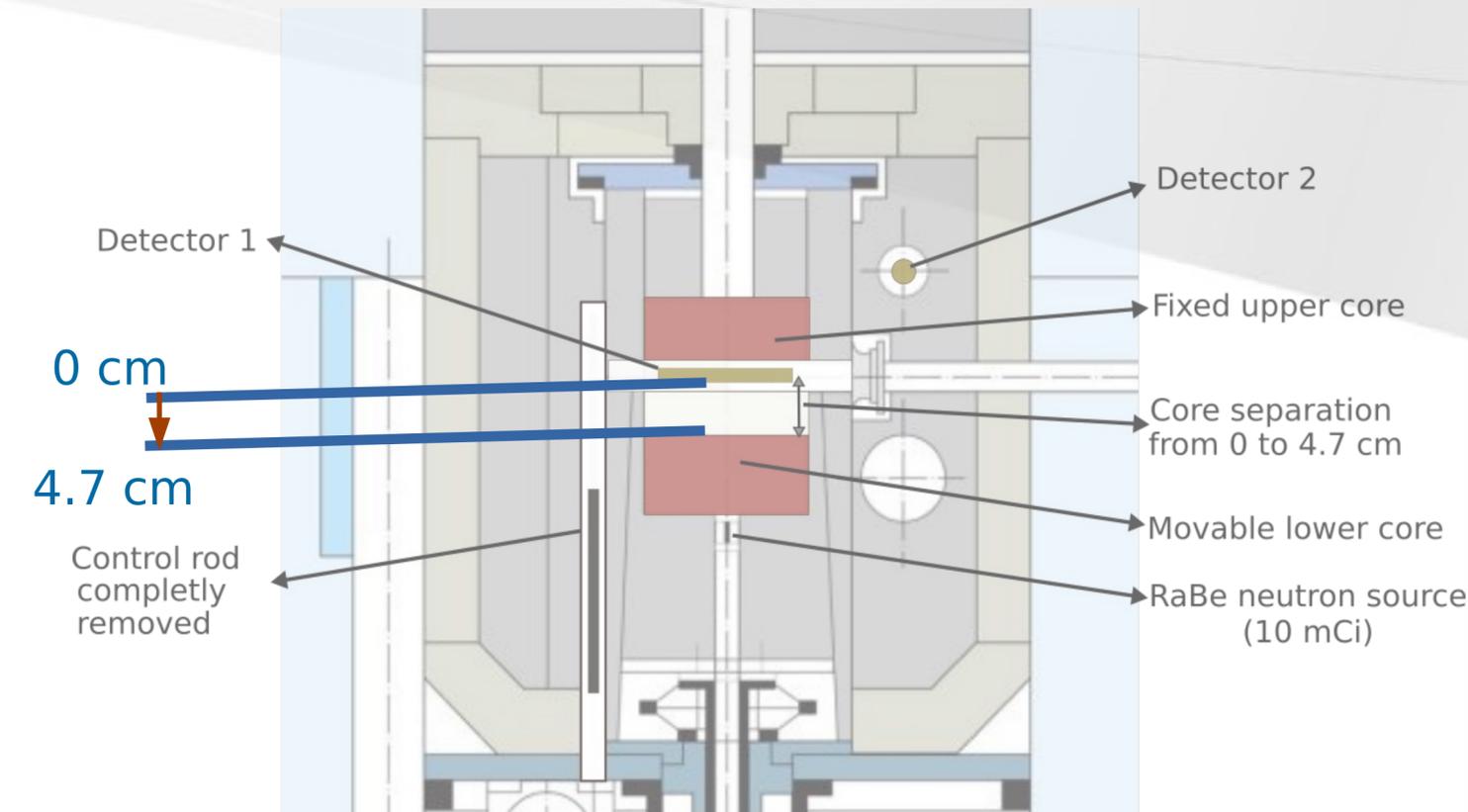
- For comparison purposes, the reactivity worth of the lower part of the core was estimated with an alternative method
- The reactor has a shut-down mechanism in which the lower core section fall by gravity in less than 100 ms (core-drop)
- As the core-drop is done between two subcritical levels, the Least Squares Inverse Kinetic Method was applied



- It allows determination of the strength of the neutron source, allowing the implementation of a subcritical reactimeter:

$$\$(t) = 1 + \frac{\Lambda^*}{R(t)} \left[ \frac{dR(t)}{dt} - \sum_{i=1}^6 \lambda_i \tilde{C}_i(t) - \tilde{Q} \right]$$

Reactimeter without the source term

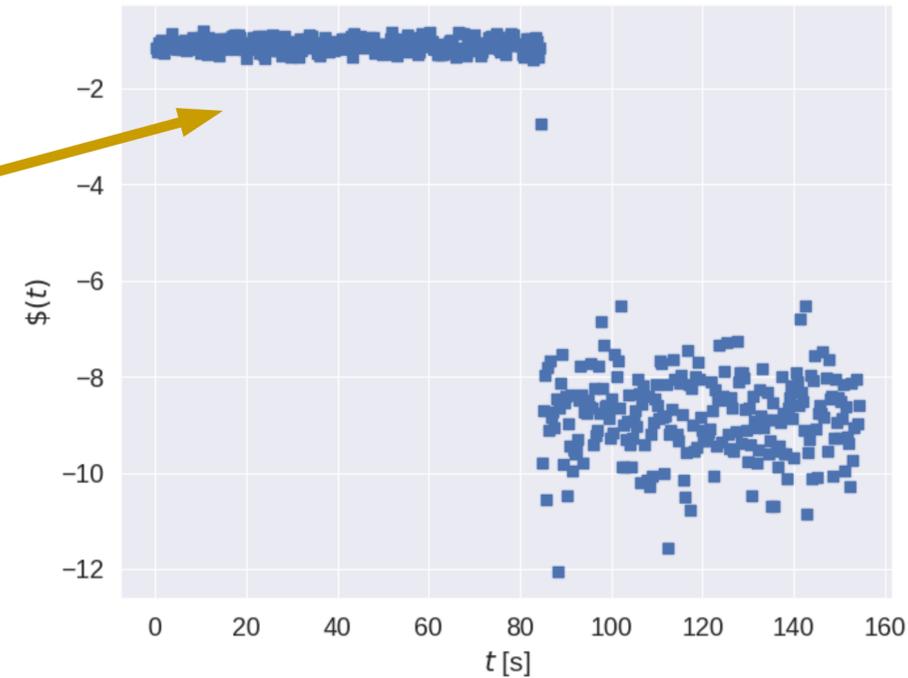
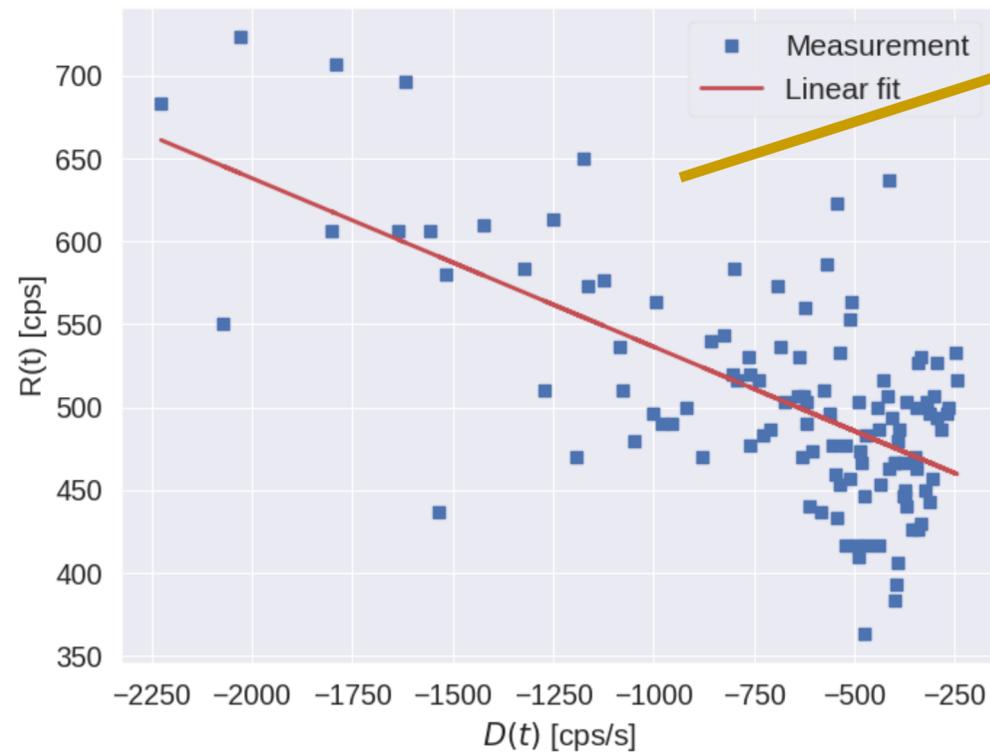


# Least Squares Inverse Kinetic Method

- Using a clever combination of variables during the transient of the core-drop, a linearized equation can be found:

$$R(t) = \frac{\Lambda^*}{\beta_f - 1} D(t) - \frac{\Lambda^* \tilde{Q}}{\beta_f - 1}$$

Source term is found



Reactimeter with the source term

- Comparison for reactivity estimations using  $\alpha$ -Feynman and LSIKM (initial separation of 0 cm and final separation of 4.7 cm)

	Detector 1		Detector 2		Reference
	$\alpha$ -Feynman	LSIKM	$\alpha$ -Feynman	LSIKM	
$\beta_i$	-1.15(2)	-1.1(1)	-1.10(2)	-1.1(1)	-
$\beta_f$	-7.5(1)	-8.8(8)	-5.4(3)	-13.6(8)	-
$\Delta\beta$	6.4(1)	7.7(8)	4.3(3)	12.5	6.83



$$D(t) = \frac{dR(t)}{dt} - \sum_{i=1}^6 \lambda_i \tilde{C}_i(t)$$

# Conclusions

- Successful implementation of neutron noise techniques in the RA-4 reactor (for the first time)
- Reduction of the data correlation of the bunching technique for a better assessment of the uncertainties in the estimated parameters
- Further analysis is needed in order to fully explain the different behaviour of detectors placed in the core and reflector regions (two regions models)
- The result of this measurement campaign could be used for validating simulations of neutron noise experiments





**Thank you very much for your attention**



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