REDUCTION IN SIZE OF BOTTOM-MOUNTED REACTIVITY CONTROL MECHANISM OF RESEARCH REACTOR FOR SEISMIC QUALIFICATION TEST

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ABSTRACT

A new research reactor employing bottom-mounted reactivity control mechanism (RCM) is under development and its safety function which is a shutdown of the reactor under earthquake events should be verified through a test. However, the real system is too heavy to be excited artificially, and hence the size of a test rig should be reduced somehow. As a preliminary study for development of a seismic test rig, this paper presents how to reduce the length of an extension shaft which is a main component of RCM while it maintains dynamic characteristics of the real system.

In this paper, instead of reduction in length of the shaft, the inner/outer radius of the shaft and water gap size between the shaft and its guide tube will be modified in order to match its natural frequency and displacement due to seismic excitation to those of the real system. Furthermore, a proper mass which does not increase the stiffness will be inserted into the hollow shaft. Then, dynamic equation was derived for the beam model and an optimization problem was defined and solved. The result shows that the design modification is reasonable for description of dynamic characteristics of the real system under earthquake events.

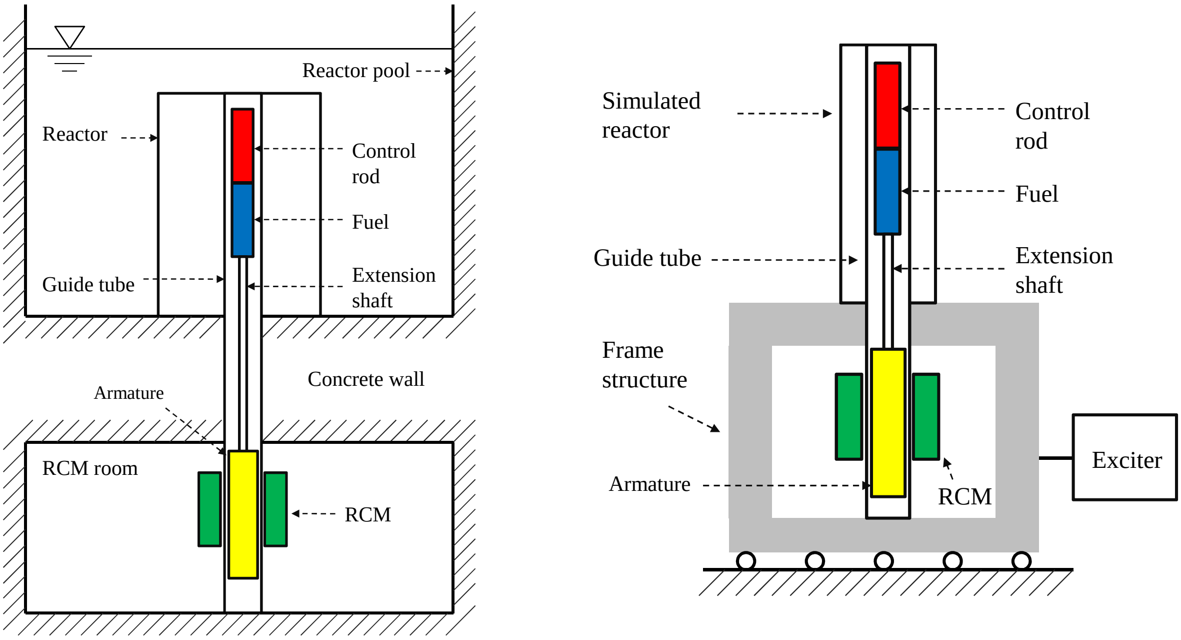
1. Introduction

A new research reactor employing a bottom-mounted reactivity control mechanism (RCM) is under development. As schematically shown in figure 1(a), the reactor is located at the bottom of the water-filled reactor pool, and its reactivity control rods are driven by the RCM located in the RCM room below the reactor pool. Although the safety function of the RCM during the earthquake events should be verified through testing, the whole seismic system encompassing the reactor, RCM, and reactor pool is too big and heavy to be excited. Therefore, a seismic test rig with a reduced size needs to be developed.

The functions of RCM are to control the reactivity of the reactor and shut down the reactor safely by a gravity drop of the control rods. The operability and shutdown function shall be maintained under an operating basis earthquake (OBE) and safe shutdown earthquake (SSE). The factors that can affect the drop time are the dynamic characteristics of the components in the reactor and a collision between the shaft and guide tube from seismic excitations. Hence, the test rig should substantially reflect these characteristics.

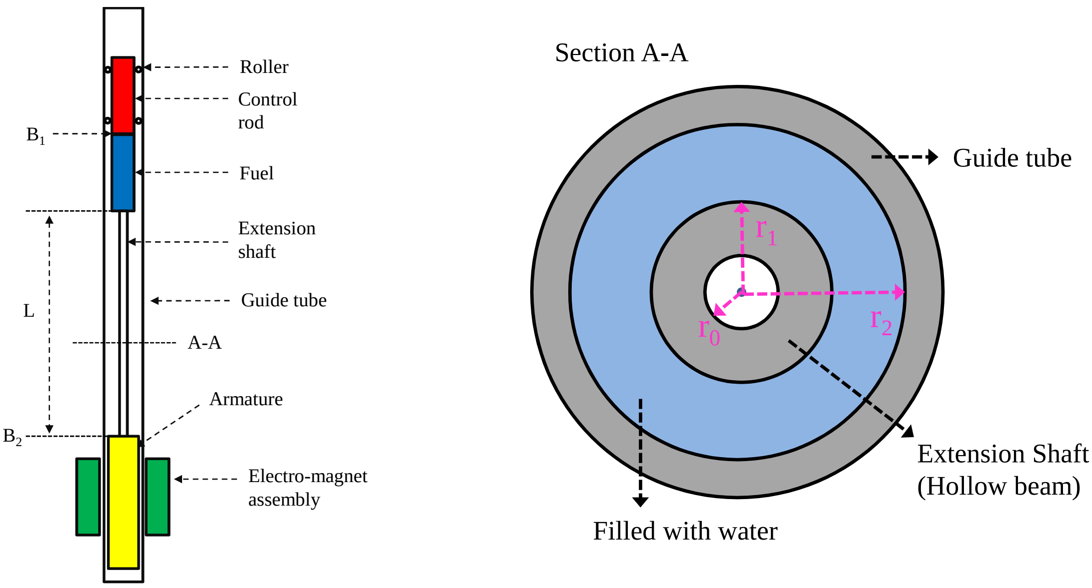
The proposed seismic test rig is described in Figure 1(b). The main differences between the real system and the test rig are a reduction in the size of the concrete wall, the length of the extension shaft, and the size of the reactor. The concrete wall is expected to be very stiff so that it can be treated as a rigid body below 33Hz, and hence, it will be substituted by a frame structure that can be stiff enough to be a rigid body. On the other hand, the natural frequencies of the extension shaft are expected to be much higher than the original system, and thus its structural modification is essential. The modification of the shaft can be accomplished through various methods, such as the insertion of a proper mass inside the hollow shaft. In this paper, it will be shown how to modify the extension shaft, maintaining the dynamic characteristics of the real system, as a preliminary research for development of a seismic test rig. Here, the dynamic characteristics of the shaft can be represented by its natural frequency and a seismic input magnitude that makes a collision between the shaft and guide tube.

In the beginning, a simplified model for the extension shaft will be presented, followed by a derivation of its dynamic equations. Then, the dynamic characteristics of the real shaft will be presented, and an optimization problem for a reduction in the size of the shaft will be defined and solved. Finally, the reasonability of the solution will be assessed.



(a) Real reactivity control mechanism (b) Seismic test rig

Fig 1. Schematic diagram of the reactivity control mechanism and its seismic test rig



(a) Front view of RCM (b) Cross sectional view of extension shaft

Fig 2. Schematic diagram of RCM

2. Simplified beam model for reactivity control mechanism

The real RCM is a quite a complicated system, and it therefore quite hard to use directly for design purposes. Therefore, a simplified model to describe the dynamic characteristics of the real system will be presented in this section.

A schematic diagram of the original RCM is shown in figure 2(a), and a cross sectional view of the extension shaft is also shown in figure 2(b). The RCM consists of a control rod, fuel, an extension shaft, an armature, a guide tube, and an electro-magnet assembly. It should be noted that the scale of figure 2(a) is not the same as the real size, and the length of the extension shaft (L) is much longer than the others. The upper end of the guide tube is connected to the reactor, the middle is fixed in the concrete wall, and the lower end is fixed to the bottom of the RCM room; therefore, the dynamic characteristics of the RCM and reactor should be considered together for a precise analysis. However, as a preliminary research, the

reactor is excluded in the model, and the guide tube is considered as a rigid wall for simplification.
There are rollers between the control rod and the guide tube so that the control rod can slide against the guide tube. Furthermore, the gap size between the armature and the guide tube is very small and it can be considered as a sliding condition. Therefore, a beam model with a fixed-fixed boundary condition at points B_1 and B_2 in figure 2(a) would be an appropriate boundary condition for the RCM. However, because the moment of inertia and the length of the fuel are much larger and shorter than those of the extension shaft, it would be reasonable to assume a fixed-fixed beam with length L.
As mentioned before, the important feature that should be considered during the drop of RCM is a collision between the extension shaft and guide tube. Therefore, a reduced model should have the same first natural frequency and mode shape as the real one because the first mode has the largest displacement and the natural frequency is related to the input magnitude of the seismic excitations. In addition, a collision must occur in the reduced model by the input magnitude which causes a collision of the real system. In this section, dynamic equations used to calculate the natural frequencies and displacement by a harmonic excitation will be presented.
A general dynamic equation for a Euler-Bernoulli beam is expressed as equation (1), and its natural frequency (ω_k) is shown as equation (2)[1].

$$\rho \frac{\partial^2}{\partial t^2} w(x,t) + d\frac{\partial}{\partial t} w(x,t) + EI\frac{\partial^4}{\partial x^4} w(x,t) = F(x,t) \quad (1)$$

$$\omega_k = \left(\frac{\mu_k}{L}\right)^2 \sqrt{\frac{EI}{\rho}} \quad (2)$$

where *w* is the displacement, ρ is the mass per unit length, *d* is the viscous damping coefficient, *E* Young's modulus, *I* is the moment of inertia, *F* is the external force, *L* is the length of the beam, and μ_1 is 4.73 for the first mode of a beam with a fixed-fixed boundary condition. Equation (1) can be rewritten using modal combinations ($w(x,t) = \sum_{k=1}^{\infty} q_k(t)u_k(x)$) as below:

$$\sum_{k=1}^{\infty} \rho u_k(x)\left\{\ddot{q}_k(t) + 2\zeta_k\omega_k\dot{q}_k(t) + \omega_k^{\ 2} q_k(t)\right\} = D(x)f(t) \quad (3)$$

where q_k denotes the modal coordinates, ω_k is the natural frequency, ζ_k is the damping ratio, and *D(x)* is spatial distribution of the external force. u_k is the eigenfunction of the k^{th} mode and expressed for a fixed-fixed beam by

$$u_k(x) = \cosh\beta_k x - \cos\beta_k x - \alpha_k(\sinh\beta_k x - \sin\beta_k x) \quad (4)$$

where $\beta_k = \mu_k / L$ and $\alpha_k = (\cosh\mu_k - \cos\mu_k)/(\sinh\mu_k - \sin\mu_k)$.

As already mentioned, the displacement of the beam is of main interest, and the first mode has the largest displacement, and thus the response of the first mode shape should be calculated. Multiplying $u_1(x)$ with equation (3), integrating from 0 to *L*, and using the orthogonality of the eigenfunctions (u_k) yield

$$\rho\gamma_1\left\{\ddot{q}_1(t) + 2\zeta\omega_1\dot{q}_1(t) + \omega_1^{\ 2} q_1(t)\right\} = \int_0^L u_1(x)D(x)dx \cdot f(t) \quad (5)$$

where γ_1 is equal to $\int_0^L u_1 \cdot u_1 dx$. By dividing equation (5) by $\rho\gamma_1$, a dynamic equation in the modal coordinate can be obtained as below:

$$\ddot{q}_1(t) + 2\zeta\omega_1\dot{q}_1(t) + \omega_1^{\ 2} q_1(t) = b_1 \cdot f(t) \quad (6)$$

where b_1 is equal to $\int_0^L u_1 \cdot D(x)dx/(\rho\gamma_1)$. When *f*(*t*) is a unit sinusoidal excitation with the first natural frequency (ω_1), the response magnitude in the modal coordinate is expressed by

$$|Q| = \frac{b_1}{2\zeta\omega_1^{\ 2}} \quad (7)$$

In a spatial coordinate, the maximum displacement of the extension shaft can be derived as

$$w_{\max} = w\left(\frac{L}{2}\right) = u\left(\frac{L}{2}\right) \cdot |Q| = u\left(\frac{L}{2}\right) \cdot \frac{b_1}{2\zeta\omega_1^{\ 2}} \quad (8)$$

Parameter	Value
Inner radius of shaft (r_0) [mm]	10.5
Outer radius of shaft (r_1) [mm]	16.5
Inner radius of guide tube (r_2) [mm]	25.0
Length of shaft (L) [m]	4
Damping ratio (ζ)	0.03
Density of shaft (ρ_s) [kg/m^3]	8100
Density of water (ρ_w) [kg/m^3]	1000
Young's modulus of shaft (E) [GPa]	205
Magnitude of ground acceleration ($\ddot{u}_g$) [m/s^2]	9.806 (=1g)

Tab 1: Parameters of the real extension shaft

3. Dynamic characteristics of real extension shaft

The real extension shaft is a hollow beam submerged in water, as shown in figure 2(b). In this case, the mass per unit length becomes [2]

$$\rho = \pi\left(r_1^2 - r_0^2\right)\rho_s + \pi r_1^2 \rho_w \left(\frac{r_2^2 + r_1^2}{r_2^2 - r_1^2}\right) \tag{9}$$

where r_0 and r_1 are the inner and outer radii of the extension shaft, r_2 is the inner radius of the guide tube, ρ_s is the density of the shaft, and ρ_w is the density of water. The second term on the left-hand side is an added mass term caused by water. In addition, the external force ($D(x)$) due to seismic excitation appears as

$$D(x) = \left\{\pi\left(r_1^2 - r_0^2\right)\rho_s - \pi r_1^2 \rho_w\right\} \cdot \ddot{u}_g \tag{10}$$

where $\ddot{u}_g$ is the magnitude of the ground acceleration. The second term in the brace is a buoyant mass term from water. Finally, the first natural frequency and maximum displacement of the shaft can be calculated by substituting equations (9) and (10) into (2) and (8) with the parameters shown in table 1. The result shows that the natural frequency is 8.86Hz, and the maximum displacement is 3.6cm. Here, the gap size between the shaft and guide tube is 0.85cm, as can be seen in table 1, and the maximum displacement is 3.6cm with the unit gravitational acceleration (=1g) input. Therefore, it can be seen that a collision will occur by seismic excitations with a 0.236g magnitude at 8.86Hz.

4. Design of reduced model

As mentioned before, the natural frequency of the modified model should be the same as the real one, and the modified one should also collide with the guide tube when the real one has a collision. However, the natural frequency of a beam is inversely proportional to L^2, and the maximum response is proportional to L^4, which means that a shorter shaft leads to a much higher natural frequency and smaller displacement than the real one. Therefore, the inner and outer radii of the shaft (r_0, r_1) and inner radius of guide tube (r_2) will be modified, and some material that has a large density without stiffness will be filled into the hollow shaft.

Because of the material inside the shaft, the mass per unit length and external force (equations (9) and (10)) become the following:

$$\rho = \pi\left(r_1^2 - r_0^2\right)\rho_s + \pi r_0^2 \rho_0 + \pi r_1^2 \rho_w \left(\frac{r_2^2 + r_1^2}{r_2^2 - r_1^2}\right) \tag{11}$$

$$D(x) = \left\{\pi\left(r_1^2 - r_0^2\right)\rho_s + \pi r_0^2 \rho_0 - \pi r_1^2 \rho_w\right\} \cdot \ddot{u}_g \tag{12}$$

where ρ_0 denotes the density of the filling. To satisfy the same natural frequency and a collision with the same input magnitude, an optimization problem, which is a function of r_0, r_1 and r_2, has been formulated as below:

$$\min_{r_0, r_1, r_2} \left[\left\{\omega_{real} - \omega_{reduced}\right\}^2 + S\left\{ \left(\frac{w_{\max}}{r_2 - r_1}\right)_{real} - \left(\frac{w_{\max}}{r_2 - r_1}\right)_{reduced} \right\}^2 \right] \tag{13}$$

where ω_{real} and $\omega_{reduced}$ denote the first natural frequency of the real and reduced model, and S

indicates the scaling factor. w_{max} is the maximum displacement from a seismic input of 1g magnitude at the first natural frequency. The ratio of w_{max} to (r_2-r_1) means the seismic input magnitude that causes a collision in unit g. A constraint equation is given by

$$0 < r_0 < r_1 < r_2 < 10\text{cm} \quad (14)$$

The optimization problem has been solved using the 'fmincon' function in Matlab with the parameters in table 2, and two possible solutions were obtained, as shown in table 3. Both of the designs show the same natural frequency as that of the real model (8.86Hz), and a collision takes place by the very similar input magnitude compared to the real one (0.236g). However, the thickness of the shaft for solution #2 is 1.2mm, which is much thinner than the real one (6mm), and thus the shaft can be easily damaged during drop tests with seismic excitations. Therefore, solution #1 seems to be the best choice.

Parameter	Value
Length of shaft [m]	2.2
Damping ratio	0.03
Density of shaft [kg/m^3]	8100
Density of water [kg/m^3]	1000
Density of filling (mercury) [kg/m^3]	13600
Young's modulus of shaft [GPa]	205
Magnitude of ground acceleration [m/s^2]	9.806 (=1g)

Tab 2: Parameters of the reduced extension shaft

	Real model	Design #1	Design #2
r_0 [mm]	10.5	2.9	8.8
r_1 [mm]	16.5	5.7	10.0
r_2 [mm]	25.0	18.7	23.5
Thickness of shaft (r_1-r_0) [mm]	6.0	2.8	1.2
Gap of water (r_2-r_1) [mm]	8.5	13.0	13.5
Maximum displacement [mm]	36.0	55.3	57.3
First natural frequency [Hz]	8.86	8.86	8.86
Input magnitude causing collision [g=9.806m/s^2]	0.2360	0.2355	0.2360

Tab 3: Possible design variables for reduced shaft model - solutions for equation (13)

5. Conclusion

In this paper, a reduced extension shaft model for a seismic test rig has been developed as a preliminary research with many simple assumptions. It shows that the length of the shaft can be reduced from 4m to 2.2m while the dynamic characteristics of the shaft are still maintained. However, very simple assumptions were used in this study, such as a fixed-fixed boundary condition for the extension shaft, a rigid wall assumption for the guide tube, and so on. In the real system, the guide tube is not a rigid body and is connected to the reactor, and thus the boundary condition for the shaft is actually not a fixed-fixed condition. Moreover, the coupling effects of the shaft and the guide tube should also be considered in the model. Therefore, a further study needs to be carried out to obtain more precise results in the future.

6. References

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[2] ASCE 4-98 (2000). *Seismic Analysis of Safety-Related Nuclear Structures and Commentary,* ASCE, Reston, Virginia.